67. $w = \dfrac{x}{y + 2z}$; $\dfrac{\partial^3 w}{\partial z\, \partial y\, \partial x}$, $\dfrac{\partial^3 w}{\partial x^2\, \partial y}$

$$\frac{\partial w}{\partial z} = x\left(-(y+2z)^{-2}\, 2\right)$$

$$= \frac{-2x}{(y+2z)^2}$$

$$\frac{\partial^2 w}{\partial z \partial y} = \frac{4x}{(y+2z)^3}$$

$$\frac{\partial^3 w}{\partial z \partial y \partial x} = \frac{4}{(y+2z)^3}$$

$$\frac{\partial w}{\partial x} = \frac{1}{y+2z}$$

$$\frac{\partial^2 w}{\partial x^2} = 0$$

$$\frac{\partial^3 w}{\partial x^2 \partial y} = 0$$

69. Use the table of values of $f(x, y)$ to estimate the values of $f_x(3, 2)$, $f_x(3, 2.2)$, and $f_{xy}(3, 2)$.

x \ y	1.8	2.0	2.2
2.5	12.5	10.2	9.3
3.0	18.1	17.5	15.9
3.5	20.0	22.4	26.1

$f_x(3, 1.8) \approx 7.5$

$$f_x(3, 2.2) = \frac{16.8}{1} = 16.8$$

$$f_x(3,2) \approx \frac{f(3.5,2) - f(2.5,2)}{3.5 - 2.5}$$

$$= \frac{22.4 - 10.2}{1} = 12.2$$

f_x

x \ y	1.8	2	2.2
3.0	7.5	12.2	16.8

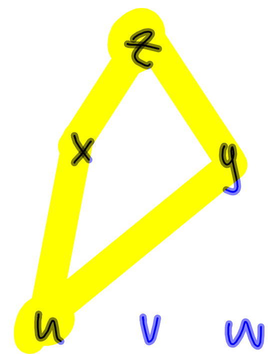
$$f_{xy}(3,2) = \frac{f_x(3,2.2) - f_x(3,1.8)}{2.2 - 1.8} = \frac{16.8 - 7.5}{.4}$$

$$= \frac{93}{4} = 23.25$$

21–26 Use the Chain Rule to find the indicated partial derivatives.

21. $z = x^2 + xy^3$, $\quad x = \underbrace{uv^2 + w^3}_{=2}$, $\quad y = \underbrace{u + ve^w}_{=3}$;

$\dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}, \dfrac{\partial z}{\partial w}$ when $u = 2, v = 1, w = 0$



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= (2x + y^3)(v^2) + (3xy^2)1$$

$$= (31)\ (1) + (54)(1) = 85$$

37. $u = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$

$$\frac{\partial u}{\partial x_1} = \frac{1}{2}(x_1^2 + x_2^2 + \cdots + x_n^2)^{-\frac{1}{2}} \cdot 2x_1$$

$$= \frac{x_1}{\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}} = \frac{x_1}{u}$$

$$\frac{\partial u}{\partial x_n} = \frac{x_n}{u}$$

57–60 Verify that the conclusion of Clairaut's Theorem holds, that is, $u_{xy} = u_{yx}$.

57. $u = x\sin(x + 2y)$ **58.** $u = x^4y^2 - 2xy^5$

$u_x = \sin(x+2y) + x\cos(x+2y)$

$u_{xy} = 2\cos(x+2y) - 2x\sin(x+2y) = u_{yx}$

$u_y = 2x\cos(x+2y)$